Experimental Evidence of Cooperation Phenomena Application to a Loudspeaker with Rub*

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An experimental investigation into cooperation phenomena is presented, which in this case can be described as the contribution of energy from a random signal to the energy of a periodic signal under certain conditions. Previously cooperation phenomena have been described mainly from a theoretical point of view. However, well-known practical examples exist. For the present experiment a loudspeaker was used that had a slightly rubbing voice coil subjected simultaneously to excitation from a random signal and from a periodic signal. Test results clearly show the influence of cooperation on both the random and the periodic signals. The random signal acquires additional "agility," allowing some of its energy to be transferred to the periodic signal. Experimental results are compared with theoretical analysis based on limit cycles and Van der Pol's oscillators. The application of limit cycles is then extended to friction-induced phenomena. The subject matter is of general interest in the dynamics of nonlinear systems and of more specific interest in the behavior and testing of loudspeakers.

INTRODUCTION

The cooperation phenomenon represents the level increase of a periodic signal due to the presence of another signal, usually a random one, within a nonlinear process or medium. This effect is also known as stochastic resonance. It is called resonance because of the similarities that exist between the curves of amplitude versus frequency in frequency analysis and the amplitude curves of the periodic signal versus the noise level in stochastic cooperation analysis. We shall show that under certain conditions noise cooperates with the periodic signal and increases its level.

We shall also see how a fine mechanism of transference of the random signal to the periodic signal generates the spectacular effect of cooperation to which we are not used to in the linear world. It should be taken into account that this transference takes place in some specific areas of the spectrum. This means that the periodic signal surrounded by the random signal produces what we might call a snowball effect. The reader probably remembers that the principle of superposition, which is true for linear systems, also applies to each of the spectral components of the total signal. This implies that the principle of superposition must be fulfilled in each and everyone of the frequencies that constitute the spectrum.

* Manuscript received 2001 October 16; revised 2002 August2.

The interest of the subject is obvious when we understand that what actually is being stated is that a noise, or a random signal, can become a periodic signal. In essence, this might be shocking in any scientific field because it would come to mean that from chaos with a "seed of order," we can increase this "order."

This phenomenon is demonstrated in detail in [1]. There we learn that, according to its authors, an important application of this phenomenon is in the study of neurons and of the nervous system, but it also applies to many other fields of knowledge, such as lasers, paramagnetic resonance, and the dithering process in electronics. In fields closer to our interests, such as electroacoustics, many applications can be observed in the mechanics of strongly nonlinear effects, for example, solitary waves or "solitons" and wave propagation in media of a periodic nature. For more details see Barratt [2].

Both [1] and [2] are analytical papers and not experimental ones, working with nonlinearity caused by elasticities which obey Duffing approaches. Nonlinear systems caused by nonlinear elastic elements are quite frequent in both mechanics and electroacoustics. But this effect is not being studied in the present paper. Instead we will focus on the cases of nonlinearity caused by systems in friction. These systems in friction are based on what is known as limit cycle dynamics, which we shall examine in more detail. As of late, interest in strong nonlinear effects has been on the increase, and this is giving rise to many studies in the field of chaos. The study of chaos and catastrophe theory is nowadays almost ubiquitous in most scientific, technical, and humanistic literature. A further work on this subject is Schroeder [3], which also gives a description of the theory of numbers and fractals. It is a very interesting and highly recommended paper for those who wish to initiate themselves in these fields.

The experiments being described have been carried out in the field of electroacoustics, but the subject is of true universality, and this grants our paper a wider interest. Nevertheless, the reader must keep in mind that however universal the subject might be, it still proves very useful in electroacoustics, a field in which linear models cannot justify certain behaviors of some of the transducer devices we manufacture and use. We understand that these are the theories that need to be explored in order to explain some of these unwanted noises that our transducers generate and which consequently render them unacceptable to clients and users with high auditory sensitivity standards.

1 PRELIMINARY CONCEPTS OF THE EXPERIMENTAL COOPERATION PROJECT

1.1 Brief Explanation of Limit Cycles in a Self-Oscillator by Friction

As mentioned before, in [1] this cooperation is attributed to nonlinearities caused by stiffness. But we propose to focus on this phenomenon from the limit cycle theory. The reader will recall that self-oscillators take energy from the power supply in such a way that they generate an alternate signal from direct current. In electronics, multivibrators, for example, work in this way. In acoustics, wind instruments take the energy that will become the vibration of the air molecules from the unidirectional flow only by means of the musician's blow. Within the field of musical instruments, those activated by a bow, such as the violin, constitute some of the most characteristic cases. A violinist draws the bow across a string, forcing it to move in one direction (except when the bow "finishes" and the violinist has to reverse). From this friction the string vibrates, sending this strength to the rest of the instrument, which in turn generates pressure that is sent to the surrounding air. This friction effect brings about a synchronized energy contribution from the bow to the string, which oscillates in an almost free movement, and this, in a synchronized manner, is what makes it take the necessary energy. Neither too much energy is taken by the string from the bow, for this would make it unstable, nor too little, since it would extinguish it. If we could see the representation of the phase plane in which the velocity of the string's stationary wave is represented versus its displacement, we would find a very stable closed orbit, which is known as the limit cycle.

In the case of oscillators, which generate limit cycles, any initial condition, external or internal, ends up falling inside the cycle. Smaller oscillations than the cycle itself increase until they equal it, and larger oscillations decrease to that amplitude. This is why it is said that the cycle is a very stable closed orbit. Those familiar with the literature on strong nonlinear phenomena should keep in mind that a limit cycle is a type of attractor.

The phenomenon of obtaining oscillating energy from a nonoscillating source is quite common, and we have all seen it in our old grandfather clocks. In these clocks the energy the pendulum looses is replaced by the counterweight, which acts as a clock winder. Before electronic watches appeared, wrist watches obtained their energy, which the oscillating system lost, from a coil spring. The strength of this coil spring went only in one direction, just like the pendulum. This situation has been known in the field of mathematics for a long time and has been studied especially in the treatises of differential equations. Jordan and Smith [4] deal with this topic in a very easy and pleasant to read way.

In the field of engineering these limit cycles have been very important in many diverse problems. When the nonlinear mechanism is caused by friction, the phenomenon of self-oscillation is known as stick, slip, and chatter [5], [6]. Den Hartog [5] is a classical and high-quality treatise on engineering vibrations, and Thomson [6] is a highly recommended and up-to-date book. A complete web page deals with the physician Van der Pol.[1]

In any case, in acoustics we refer to this kind of oscillators repeatedly. A simple example would be the squeaking sound of a poorly lubricated hinge when opening or closing a door. This noise is caused by the vibrations of the male and female elements that make up the hinge. These elements move freely, just like the violin strings. The door, like the bow, moves only in one direction (dc), and the elements of the hinge, like the string, oscillate (ac).

1.2 Proposed Approach

In the present paper we will introduce some variations of the oscillator mentioned. In the first instance, what the oscillator experiments of Van der Pol's limit cycle and ours have in common is the rub we will generate between the voice coil of a loudspeaker and a solid body in the air gap. In order to do so, we glued the spider and the voice coil to the frame slightly tilted so as to cause friction between the voice coil and the static parts. Once the rubbing system had been created, and just as we have been doing with the element of static and potential energy contribution, we needed to make the loudspeaker work in an excited way and at a specific frequency. We are now no longer working in a way analogous to Van der Pol. By sending a periodic voltage to the loudspeakers we were creating the same effect as if we had an oscillator as an external energy source. If by rubbing we made our loudspeaker oscillate due to an external periodic signal, it would be just as if the aforementioned grandfather clock were excited by another pendulum instead of an almost static counterweight, which is what gives it the potential energy.

To achieve the cooperation effect we are studying, we need to mix two very different types of signal in our nonlinear system, namely, a periodic signal and a random or noise signal. In order to mix these signals, the easiest way

[1] See, for example, the following address: http//www.exploratorium.edu/turbulent/CompLexicon//vanderpol.html.

would be to use an electronic adder, and then amplify the sum of both the periodic signal and the noise signal. Finally we would send it to the nonlinear loudspeaker, which we have already prepared for this purpose. There should be no doubt regarding the linearity and the fidelity of the electronic system, which adds the capability of generating cooperation in the loudspeaker. Therefore it is advisable that we make sure that any addition effect is of a purely electroacoustic nature and that the signals we are going to mix have not had any other kind of relationship besides the electroacoustic one. For this same reason the experiment was initially set up without including an electronic adder.

We devised a system of two degrees of freedom consisting of two loudspeakers of the same type, coupled face to face and sealed against a supporting annular ring, which allowed us to pass the microphone cable through it. Fig. 1 illustrates this arrangement. Only a small volume was left between the two diaphragms in order to achieve good interaction between both loudspeakers.

The side compliances are the compliances of the individual loudspeakers. The compliance of the confined air between both diaphragms forms the central compliance. The two mobile parts of the loudspeakers constitute the masses or inertias. Initially each loudspeaker had a different amplifier to avoid any crosstalk between channels. Later we shall see how although we maintained this two-degrees-of-freedom disposition, and both loudspeakers coupled face to face, we did use an electronic adder since it made it easier and did not cause any further problems. This setup was also maintained in order not to change the frequency of excitation of the periodic signal.

On the other hand, other experiments obtained positive results when mounting both face to face loudspeakers with a similar rub and having different amplifiers for each loudspeaker so as to avoid any interaction. As the reader may imagine, the difficulty of manufacturing two loudspeakers with the same rub prevented their use. Notwithstanding, with one of the loudspeakers carrying a periodic signal and the other a random signal we obtained very substantial cooperation. Fig. 2 shows a two-degrees-of-freedom system of the two loudspeakers and the cavity between them.

1.3 Details Concerning Loudspeaker with Rub and Measuring Equipment

To create the loudspeaker with rub we used a loudspeaker model AS 4B from Acustica Beyma. This 4-in (10-mm) 8-Ω loudspeaker had the following parameters: power 50 W, sensitivity 90 dB, frequency response 70–17 000 Hz, diameter 100 mm, Bl = 7.7 N/A, moving mass 5 grams, and resonant frequency 130 Hz. We chose this loudspeaker because it was easy to manipulate, simple to seal, lightweight, and because it had enough stroke to create friction in a zone that was not too short. The adder used was a high-fidelity electronic mixer.

Once the device was mounted as a two-degrees-of-freedom system, we obtained frequencies of 114 Hz as a first natural frequency (both loudspeakers moving in phase) and 314 Hz as a second natural frequency (both loudspeakers moving in antiphase). The experiments to be described were carried out using this configuration. Unless specified otherwise, the signals were mixed in the adder before being sent to the amplifier and the loud-

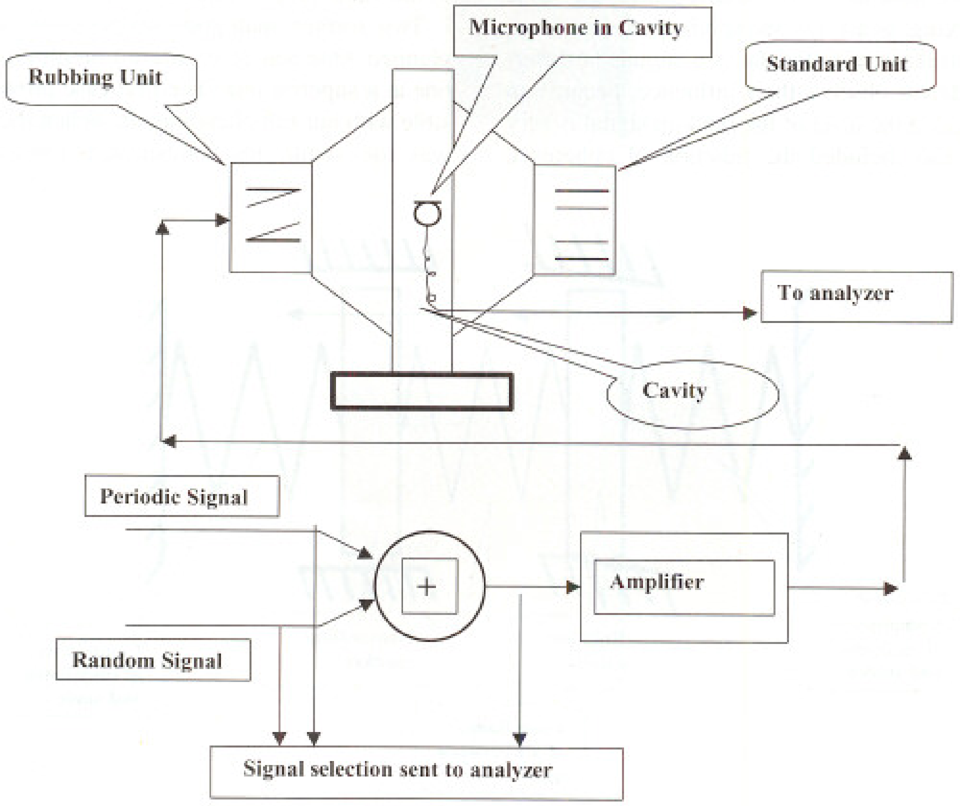

Fig. 1. Main layout for experiments, showing loudspeakers mounted face to face.

speaker with rub. The natural frequency of 314 Hz was chosen as the main frequency for the periodic signal.

The microphone used was a prepolarized electret from Microtel. It was small in size, and its wires went through a small hole in the annular ring between the two loudspeakers. We noticed later that it was not necessary for the system to work on the second natural frequency, and even then cooperation was obtained when we used direct current to block the loudspeaker with no rub.

The analyzer used throughout the study was a B&K 2035, as seen in Fig. 1. Most tests were done using a Flat Top time window, which gives a lower number of amplitude errors, and which broadens the width of the spectral lines. But this is not a main issue. In Bendat and Piersol [7] we can read further on dual-channel spectral analysis as used in our experiments. In these analyzers the input channel, or channel A, is usually the stimulus signal, and channel B is the channel that measures the output or response. In our case we usually used as stimulus the signal that attacks the power amplifier. As a response we used the sound pressure in the two-degrees-of-freedom system with the microphone introduced inside the cavity.

2 COOPERATION EXPERIMENTS AND RESULTS

In order to study the addition of noise to the signal, we applied the periodic signal to the adder, which in this case was a 314-Hz frequency sinusoidal signal. Then we sent it to the amplifier and the loudspeaker with rub, and we were able to see the sound pressure response in the cavity. At the outset the random stimulus was white noise, but further on we used pink noise. We shall mention this again when the change takes place. Finally, we injected both signals to the adder simultaneously and repeated the same operation, observing, as always, the results in the analyzer.

We carried out these tests for the two signals at different levels in order to observe their influence, because in stochastic resonance the level of the random signal is very important. We also included the function of coherence since, as in the spectral analysis of linear systems [7], it is very important to control both the operator's and the analyzer's errors. Also, this function helps us observe nonlinear effects.

Fig. 3 shows the cooperation effect between the aforementioned signals for relatively low pressure levels in the cavity. The figure has been divided into three parts to facilitate understanding. Fig. 3(a) shows the periodic signal when operating alone, Fig. 3(b) shows the random signal alone, and Fig. 3(c) shows both signals acting simultaneously. Observe the cursor in Fig. 3(a), illustrating the injection of a sinusoidal signal alone to the rubbing loudspeaker, which reads 90 dB of sound pressure at 314 Hz. Notice that although the harmonics of the sinusoidal signal are not visible in the figure, they are quite coherent. The fundamental frequency is absolutely coherent. Observe the unity value in the right lower cursor.

In Fig. 3(b) we can follow how when noise attacks only the rubbing loudspeaker, it gives a sound pressure level of 87.6 dB at this particular frequency (314 Hz). We also show that the coherence is inferior to the previous one [Fig. 3(a)]. Between input and output there is a coherence of 633 milliunits.

In Fig. 3(c) we observe what happens when a loudspeaker with rub is attacked by two simultaneous signals. It gives a sound pressure level of 100.8 dB at 314 Hz, a very substantial increase caused by this combined action. In other words, we achieve the desired effect of cooperation.

Take into account that at this level of both signals, coherence makes up the greatest part in each of the spectral partials that compose it. But this is not the case for the second harmonic of the periodic signal, which at first was 0.8, but when executing the combined action it decreased to the value of the random noise.

Two further analogous cooperation experiments were planned. One was at an inferior amplitude level, the other one at a superior one. The first case proved to be not feasible with our rub phenomenon. When the amplitude level was too small, the loudspeaker jammed. The second

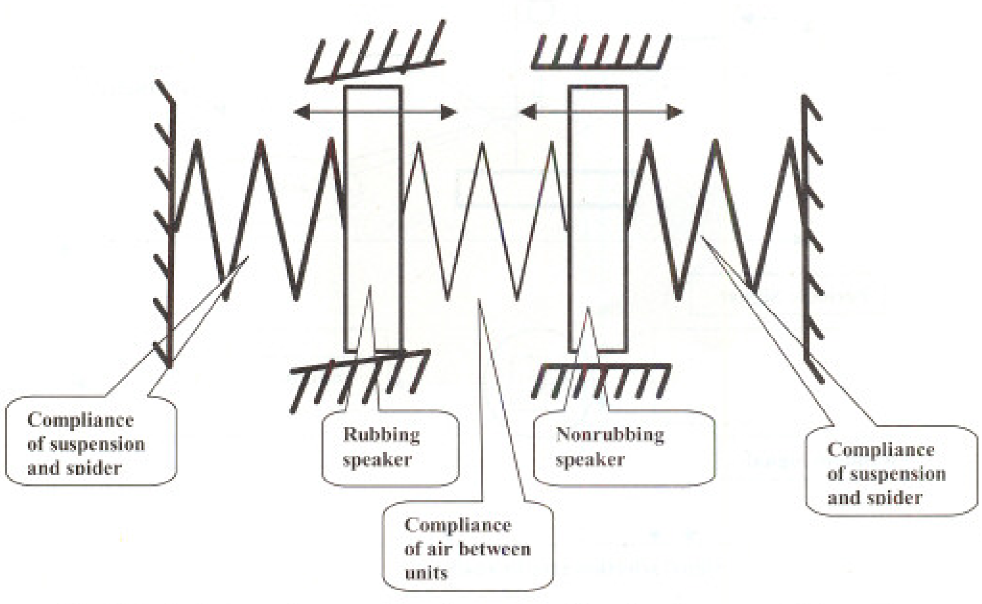

Fig. 2. Equivalent two-degrees-of-freedom system of loudspeakers mounted in a cavity.

experiment, with a higher amplitude level, was absolutely feasible and positive, producing an even higher level of cooperation than the one shown in Fig. 3(c). The results of mixing the two signals with a higher amplitude level in a loudspeaker with rub are illustrated in Fig. 4.

Just as in the previous case, Fig. 4(a) shows the

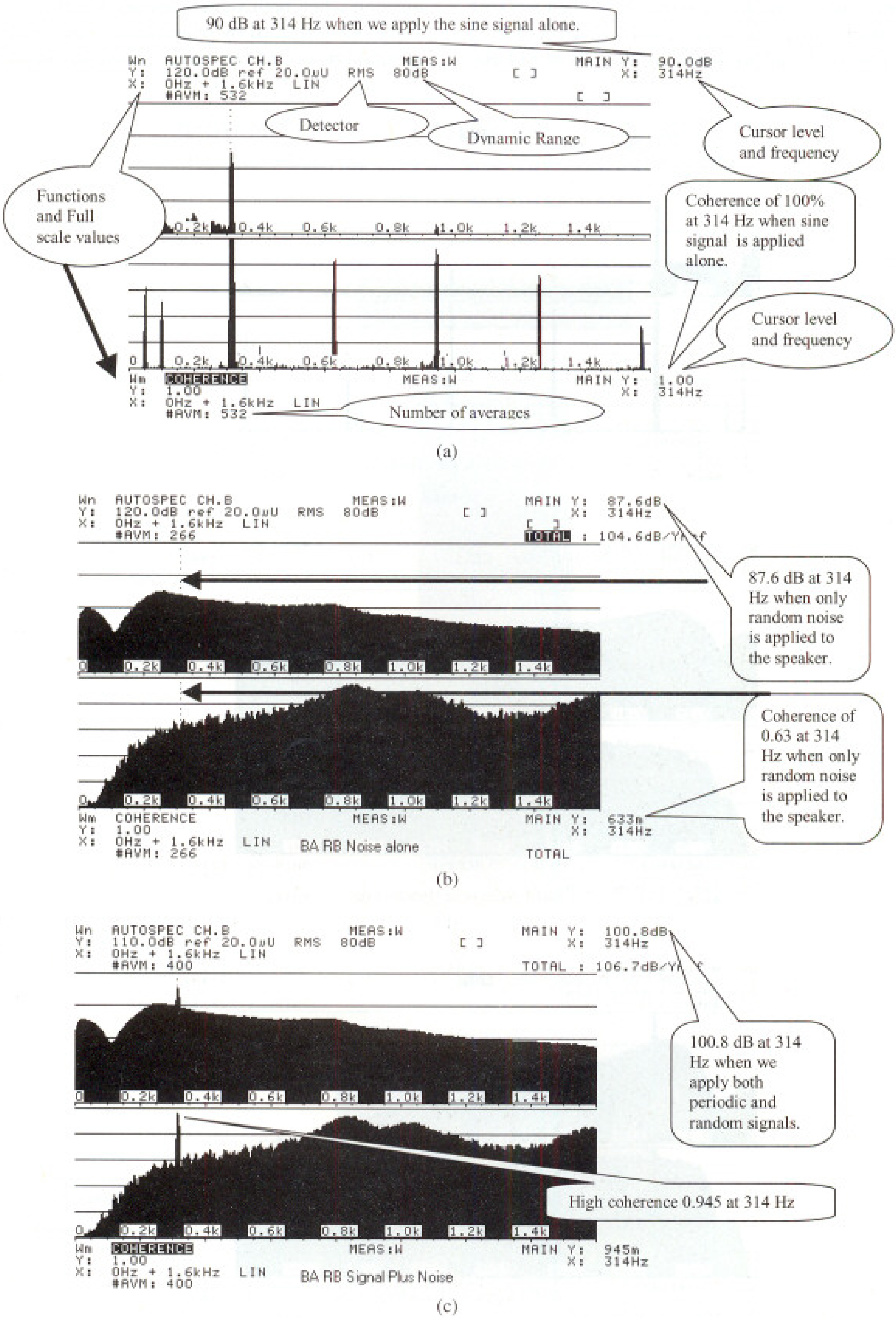

Fig. 3. Tests with rubbing loudspeaker at low level. Autospectra (spectral average) corresponding to acoustic pressure in cavity (top) and coherence between electronic signal applied to amplifier input and acoustic pressure output (bottom). (a) 314-Hz sine signal applied alone. Note that coherence is high not only for the 314-Hz component, but also for its harmonics. Outside this, coherence is near zero (bottom). (b) Random noise applied alone. Note that coherence values are frequency dependent (bottom). (c) 314-Hz sine signal and noise applied together. See cursor at 314 Hz and amount of cooperation given (top). Note that coherence at 314 Hz is near value obtained when periodic signal was applied alone (bottom).

response of the sound pressure of the sinusoidal signal alone in the cavity. Notice that the full vertical scale reads 110 dB whereas the amplitude of the frequency we are working with is 95.5 dB. Note also that coherence is high for the harmonics of the attack signal. But when working with the random signal alone [Fig. 4(b)] we tried to keep an analogous level, and we obtained 96.5 dB at 314 Hz. The coherence for this signal has an even higher level than in the previous experiment, but it is less than 1. In Fig. 4(c) we observe a level increase produced by the interaction of both signals, reaching 116.6 dB. Notice the coherence function when both signals operate together. It is similar to the one produced by noise acting alone, except for the spectral line of 314 Hz, where coherence is almost 1.

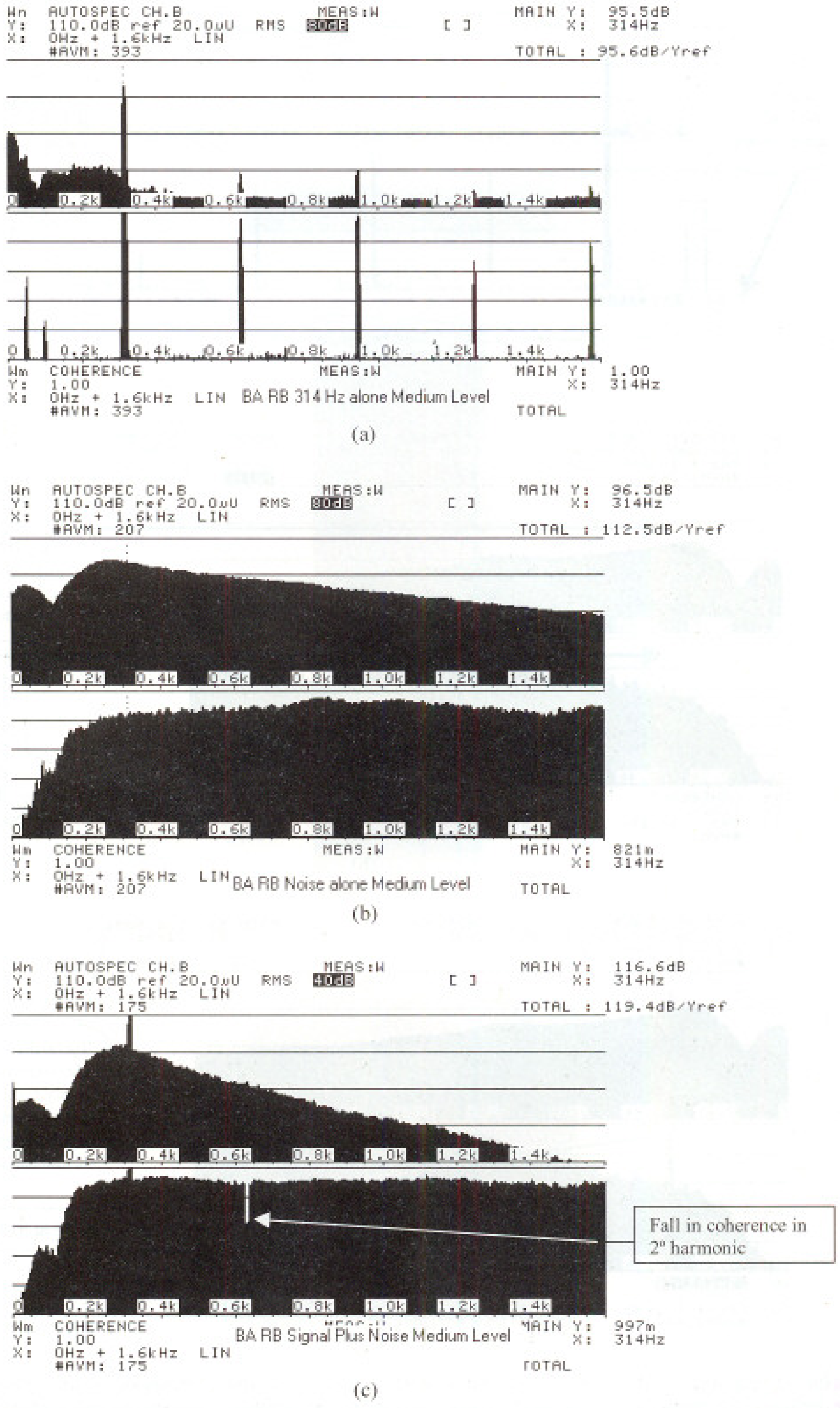

Fig. 4. Tests with rubbing loudspeaker at medium level. Autospectra of acoustic pressure in cavity (top) and coherence between acoustic signal and electronic signal from amplifier input (bottom). (a) 314-Hz sine signal applied alone. (b) Pink noise applied alone. (c) 314-Hz sine signal and noise applied together. See cursor level of 116.6 dB at 314 Hz indicating amount of noise "converted" to periodic signal (top). Note coherence at main frequency of 314 Hz and hole at its second harmonic (bottom).

The hole created in the second harmonic of 314 Hz has decreased from 980 milliunits to 600 milliunits. This is very important, for it indicates that the mechanism of transference of the random signal to the periodic signal is producing a link through a frequency higher than the principal frequencies and involving the first harmonic.

The reader should notice that it is of utter importance to detect how this transference takes place in order to fully understand what is happening. But we shall come back to this further on.

2.1 Test on a Linear System—Fulfillment of the Superposition Principle

We need to observe how the superposition principle is satisfied in a linear system. In order to do so, we need to substitute the rubbing loudspeaker by an unfaulty unit of the same type. We then submit it to the same test, under the same conditions as in the previous test. The linear loudspeaker is then tested at a high level of excitation, and it shows its linearity, thus satisfying the principle of superposition, as expected. Fig. 5(a) shows the results of our experiment. The periodic signal alone worked at sound pressure levels close to 100 dB and at a frequency of 314 Hz. Fig. 5(b) illustrates that the level used is close to 100 dB and that, as can be expected, coherence is very high in almost the entire frequency range because the system we are using is very linear.

In Fig. 5(c), which corresponds to the two signals applied simultaneously, we can observe how their addition has produced a value very close to the expected 103 dB. The cursor indicates 102.8 dB, which is a very reasonable laboratory deviation for a linear system.

It should be taken into account that we usually work with an 80-dB display dynamic range and that in this case we used 20 dB in order to obtain better display resolution. This is a comforting result because it shows that we cannot see nonlinearity where there is none.

Fortunately the world in which we live has a large number of linear components. As an interesting fact, and moving a little outside the technical world, we could say that on some rare occasions (nonlinear) we can obtain some benefits from falsehood, but in general we only obtain benefits in ordinary and correct situations (linear), from truth. Cooperation is a strongly nonlinear phenomenon, but if we can obtain some benefit from it in specific circumstances, it only seems reasonable to do so.

2.2 Complementary Measurements in Cooperation

Additional measurements of cooperation were taken to ensure that the results were reliable. First we measured the acceleration of the membrane of the loudspeaker with rub. This was done with the help of a miniature accelerometer. Results are not shown to avoid an unnecessarily long paper, but it is useful to know that they were totally analogous to those obtained with the microphone inside the cavity. Cooperation was obtained with sound pressure level increases as high as 16 dB and higher.

When measuring the membrane's acceleration we even applied higher amplitude levels in the periodic signal than in the random signal, and these results were still obtained. To study whether the phenomenon occurred only when using the frequency of 314 Hz in the accelerometric measurements, we changed the frequency of the periodic signal. We chose the frequency of 214 Hz, which was different from the two natural ones we had used already. The results given in Table 1 show that the effect was also achieved when we changed the frequency.

Using the cross spectrum, or the frequency response function, we obtained the phase angles between the electric signals of stimulus and response. The results are recorded in Table 2.

First we notice that the linear system has a unique phase angle in the three circumstances with only a one-degree error. Second it is interesting to see that the phase in which there is cooperation is close to the one with noise only. Nevertheless, these angles are dependent on the signal amplitude. For these tests, and other additional ones, we find a tendency of the random signal and the cooperated signal to have a phase lead. It all occurs as if the sinusoidal signal alone belonged to a "fly wheel." The random signal would have less inertial effect, and the cooperated or total signal would have even less.

This seems to indicate that the limit cycle takes the random signal to a phase lead, which seems quite logical since it is this signal that will be dragged into the periodic signal, consequently increasing its level. Therefore, as can be expected, the limit cycle caused by friction modifies the state of the random signal and bestows it with "agility" so that it can transfer itself in an ordered way and become part of the periodic signal's spectral line. The cooperated signal stays in this state of phase lead, which is even stronger than that of the random signal. Perhaps this new phase lead favors new transferences, which we have not tested. That is to say, it could be that the total or cooperated signal is in a situation such as to manipulate itself again in an addition sense analogous to what we have just seen. Our experiments of measuring the accelerometer signal also confirm these phase lead trends.

2.3 Study of Natural Frequencies

2.3.1 Excitation of Loudspeaker with Rub Using Pink Noise

When we study a system which we suspect may have behaviors analogous to those involved in limit cycles, it is of utter importance to know the transitory behavior because the frequencies of these elements usually mark these processes.

We already mentioned that when we hear a door creaking, the noise is marked by the frequencies of the elements that include the hinge. In our experience it is no easy task to know the position of the frequencies common to the cylinder that makes up the loudspeaker moving coil, nor those of the polar piece. Notwithstanding, we must carry out experiments that give us the main natural frequencies. Strong nonlinear systems have particularities in their behavior which are dramatically different from those present in linear systems when we try to find their eigenfrequencies. One of these idiosyncrasies is low measurement repeatability, meaning that when a system is excited by different procedures, we do not necessarily obtain the same results. When we change the level of excitation, we still obtain various results. These systems are very sensitive to the initial conditions. It also implies than when we perform a very similar excitation process, but not identical to the previous one, our results will again be different. The reader will probably remember that in the study of linear systems these circumstances do not take place in any of the aspects mentioned. Further details in this regard are given in [8]–[10]. A reference book on strong nonlinear systems [11] is much more specific than the others. In order to increase the spectral content of low

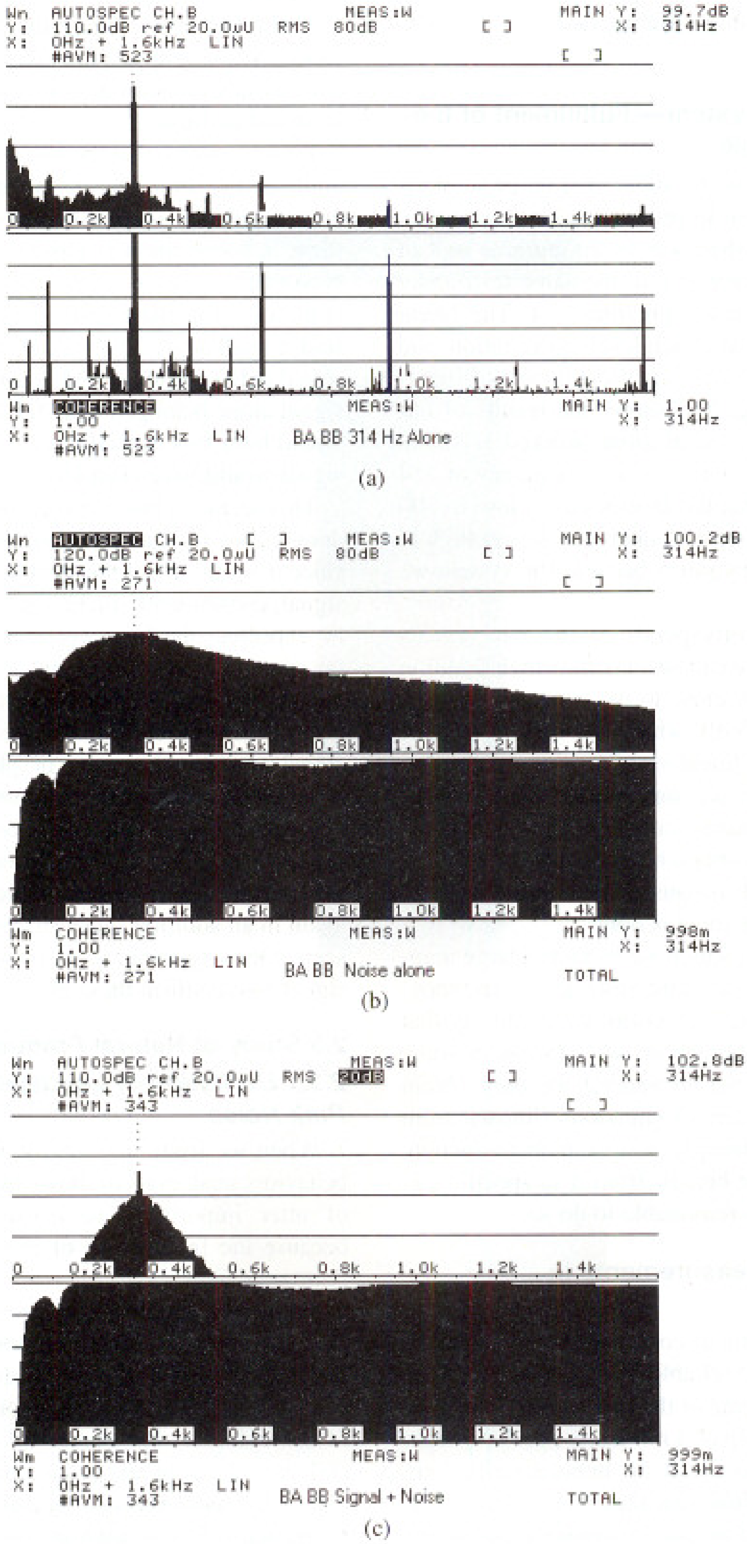

Fig. 5. Tests with nonrubbing loudspeaker. Autospectra of acoustic pressure in cavity (top) and coherence for standard AS 4B Beyma loudspeaker (bottom). (a) Periodic signal alone; cursor level 99.7 dB at 314 Hz. (b) Pink noise alone; cursor level 100.2 dB at 314 Hz. Note that system is almost linear and hence coherence is high in entire frequency range (bottom). (c) Periodic signal and pink noise together; cursor level 102.8 dB at 314 Hz. Note that coherence is very similar to case of pink noise alone, as it must be for a linear system (bottom). To show better details, dynamic range was changed from 80 to 20 dB.

frequency, we excited the rubbing loudspeaker with a pink-noise random signal instead of a white one. Working with the acceleration of the membrane and exciting it with this pink spectral shape, we observed that a peak around 740 Hz would either show or hide itself as we repeated the test. In any case, it seems reasonable to think that the natural frequency of the sensor over its fixture could be in this region. Unfortunately the natural frequencies of the elements in friction may also be in this spectral region. It is interesting to note that the sound pressure, above the natural frequencies of the units, is a consequence of the diaphragm's acceleration. And despite having modified the system because of the use of an accelerometer, the spectral acceleration shape measured in the center of the dust cup of the rubbing loudspeaker was very different from the shape of the spectral pressure in the cavity.

In the upper part of Fig. 6(a) we see the membrane acceleration and in the lower part of the cavity pressure for the already mentioned pink signal. In fact, the spectral range between 600 and 900 Hz is actually very rich for the acceleration signal, but not so for the pressure signal. This is probably due to the fact that some local eigenfrequencies of elements in friction are in this spectrum region (600 Hz to 1 kHz), and this is manifested mechanically and is less evident acoustically.

The technological limitation of not having access to an extrasmall accelerometer, such as 0.1 gram, prevents us from being able to glue it to the coil and studying the system in more detail. If these extremely small accelerometers did in fact exist, they would have very fragile cables and very low sensitivity. In order to see this variability of the results with repeated tests, observe in Fig. 6(b) how although the individual results have already been averaged, they are still quite different. Fig. 6(c) reinforces what we just said about how a periodic excitation in a loudspeaker with rub produces as a response an acceleration in the area with the alleged local natural frequencies of the elements in friction. Therefore we could conclude that when the excitation in a loudspeaker with rub is pink noise, there is a great difference between the spectrum of the pressure and that of the acceleration membrane. Also the results of several tests vary greatly. But this behavior would hardly surprise someone who is familiar with strong nonlinear systems.

2.3.2 Excitation of Loudspeaker with Rub Using a Step Function

Because the system we are working with presents the aforementioned variations, it seems advisable to perform additional tests of this nature. This is why we applied signals to see its transitory response and to observe its variability with respect to these signals. A system with rub, such as the one we are presenting, contains important irregularities. Therefore we placed an accelerometer in the loudspeaker with no rub and another one in the loudspeaker with rub. The sensor in the rubbing loudspeaker goes into the analyzer to channel B and the linear loudspeaker goes to channel A. We applied a step voltage of 4 V to the linear loudspeaker, but always respecting the initial assemblage of the two units coupled face to face. The loudspeaker with rub would be dragged by the linear loudspeaker. In the different experiments illustrated in Fig. 7(a) we can see the shape of the response for a single (nonaveraged) step voltage. In the linear system, with all the regularity of the figure, we are led to think what we already know, namely, that the device is linear. On the other hand we assume that the resonance frequency of the mounted sensor is around 1052 Hz.

If we look carefully at the lower figure, we can see that the dragging is in a low frequency, and the peak, where the cursor is, is in all probability one of the natural frequencies of some element in friction or rub, such as the polar piece, the top plate or the axial modes in the voice coil. When the experiment is performed again in exactly the same way, as shown in Fig. 7(b), it gives us a behavior with low repeatability, especially in the spectral area assigned to the natural frequencies of the elements in friction between 800 and 900 Hz. All this is analogous to what we find in a normal chatter system, although the system we are using is more complex. Further bibliography on this topic can be found in [12], [13]. We insist on the

Table 1. Cooperation levels for membrane acceleration.

Excitation periodic signal of 214 Hz (Random noise identical for all tests)	Level of Vibration of Membrane, Measured at 214 Hz		
	Periodic Signal Alone	Random Signal Alone	Periodic and Random Signals Combined
	98.2 dB	75.4 dB	107 dB

Table 2. Cross spectrum phase angles.

	Phase Angle, Periodic Signal	Phase Angle, Random Signal	Phase Angle, Both Signals Added
Linear loudspeaker, no rub	270°	−89°	−89°
Loudspeaker with rub, level of signal average	2.6°	−59.2°	−64.3°
Loudspeaker with rub, level of signal high	2°	−75°	−77°

notion that if it were possible to obtain smaller sensors, we could see perfectly how the coil oscillates when it rubs as a string or membrane would do. This rubbing is what in all probability generates the phase lead that we observed.

2.4 Visualization of the Time Signal—Cooperation in a Transient Regime

Another experiment will allow us to observe the time signal when cooperation is generated. In the literature a

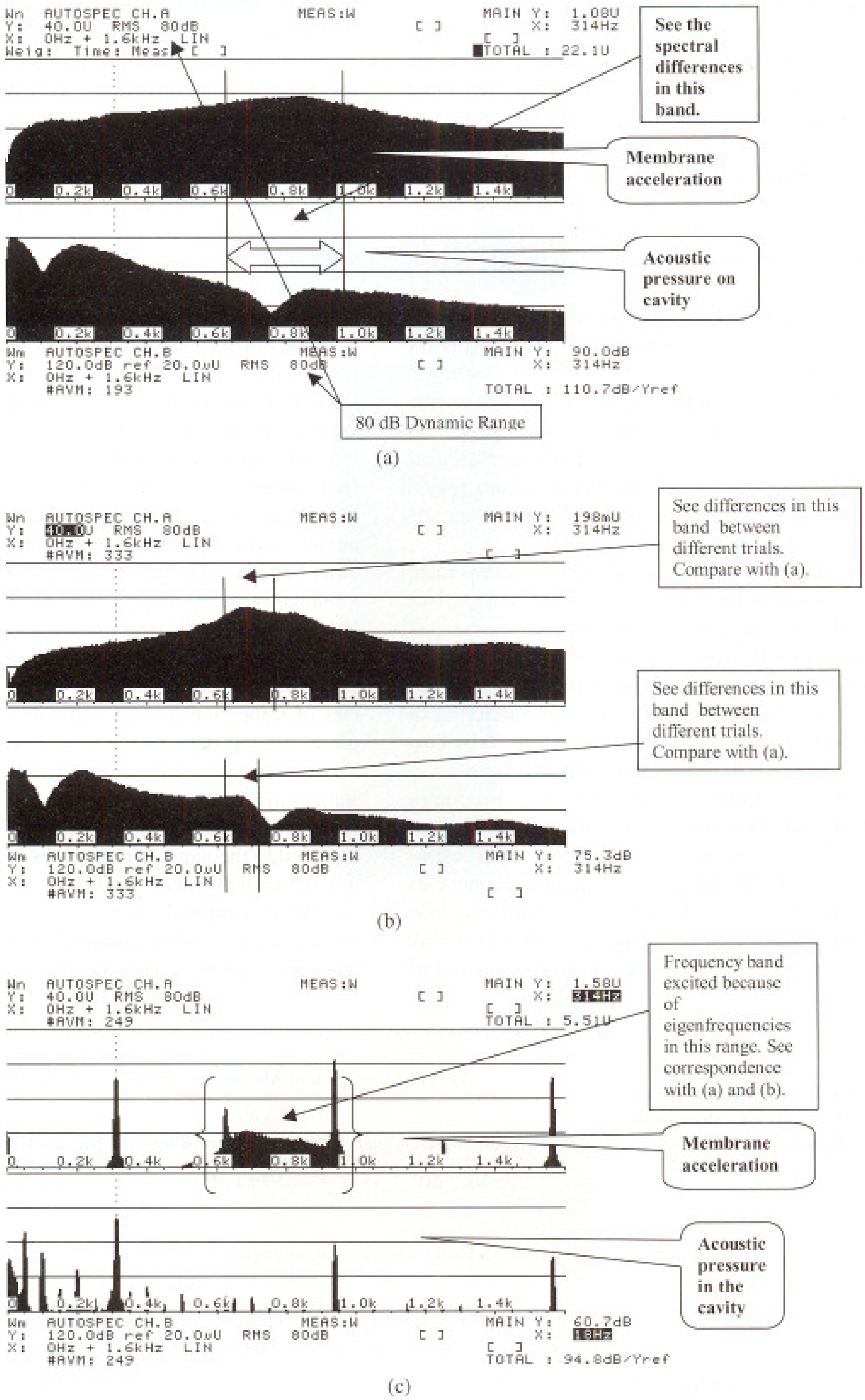

Fig. 6. Tests with rubbing loudspeaker. Autospectra of membrane acceleration (channel A) and acoustic pressure in cavity (channel B). (a) Application of electronic pink noise. Within arrow, acceleration is in a mountain, pressure in a valley. (b) Application of wide-band noise. Note that response does not have exactly the same results as in Fig. 6(a). This behavior is common in nonlinear devices or processes. (c) Application of sine wave. Note how acceleration shows eigenfrequencies, even when excitation is far from this frequency band. This behavior is highly nonlinear, in conformance with the rubbing process.

double limit cycle is sometimes mentioned as, for example, when one is inside another. We thought it interesting to observe transitions in order to note whether noise behaves like an impulse that takes vibration to a higher state. To do so, we simply injected the periodic signal into the rubbing loudspeaker, as usual. Then, using a switch, we applied the random signal for a few milliseconds and then removed it quickly. In this experiment the membrane acceleration analyzer was in channel A and the sound pressure of the cavity in channel B. We applied the periodic signal to the loudspeaker with rub. Fig. 8(a) shows its pressure and how the level decreases when we add and remove the noise. When only the signal remains after the noise has been removed, its level is higher than before.

We did not analyze how long it took for the remaining higher signal to dampen. In any case, the fact that the process of cooperation is fast seems very logical if we take into account the previous phase lead. In a further test we removed the random signal from the total signal to make it evident that the periodic signal always remains in a high state. Hence in a transitory regime cooperation occurs and, as we proved, it is a quick and agile process.

2.5 Excitation with a Squared Signal instead of a Sinusoidal Signal and Time Average

Since we performed the entire experiment with a sinusoidal signal, as far as the periodic wave goes, it seemed obvious that we should examine whether our system would also run properly with other periodical waves that were not sinusoidal. This holds especially if we take into account that it seems as though the harmonics of the sinusoidal signal are related to the process we are studying insofar as the coherence signal has decreased steeply in the even harmonics.

Immediately after having observed this phenomenon it is interesting to acknowledge how it forms and to detect which are the lock-in and synchronization mechanisms. It seems convenient to average the signal in the time domain rather than the frequency domain. These averages are sometimes called enhancement signals. Literature is abundant in these synchronized mechanisms, but further studies are required since many self-synchronized systems still cannot be explained. The physician Huygens was a pioneer in these studies on syn-

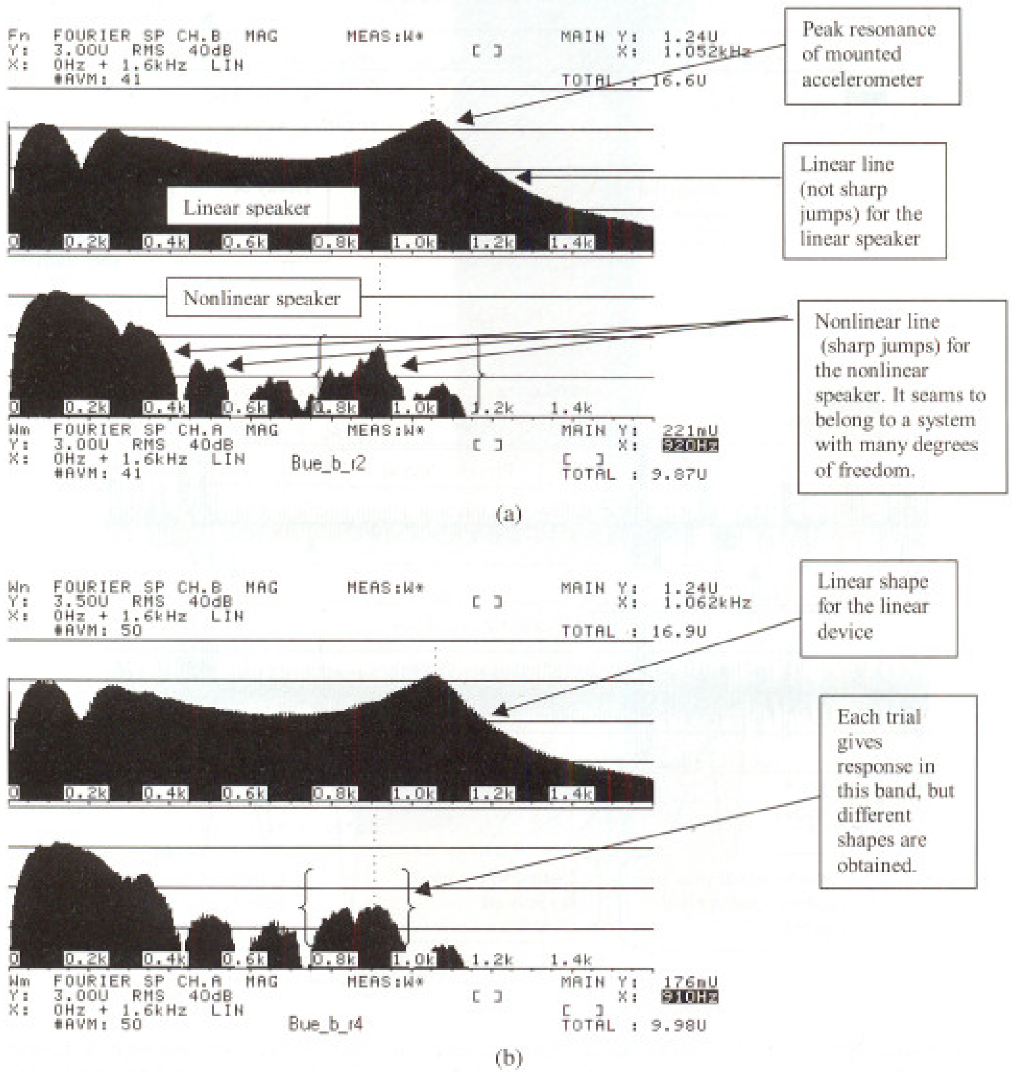

Fig. 7. Additional information on eigenfrequencies. Excitation +4 V dc, applied to linear loudspeaker. Acceleration Fourier speectra (not average or single spectra) comparing responses of rubbing (channel A) and linear (channel B) loudspeakers. Frequency bands (in brackets) are different for each trial. (a) First trial. Note nonlinear and linear behavior of each unit. (b) Second trial. Note high repeatability of linear unit.

chronism. He explained in great detail the synchronism of pendulum clocks that were connected to each other through the wall on which they hung.[2] Another easy and valuable reading is Bak [14].

Moving on, as we said, we stopped using the sinusoidal signal. Instead, as we were trying to achieve greater harmonic contents, we chose the squared wave. We worked in the time domain, using the time average mode of the B&K 2035 analyzer. In this way we can see things even more sharply for only the signals that are temporally synchronized are being averaged. For example, any signal or noise that is not synchronized is canceled. The acoustic noise in the laboratory or interferences from the mains, for example, are canceled when using time averaging, whereas with spectral averaging these noises and interferences are included. This process forces us to use a clock or synchronizer signal in the analyzer that computes the average. One square wave is used as a synchronization clock. This clock signal (inside the analyzer) is used to identify the first sample of each data block to be averaged. This first sample is

[2]See, for example, http://unisci.com/stories/20003/0911003.html and http://www.sciencemuseum.org.uk/collections/exhiblets/huygens/start.asp.

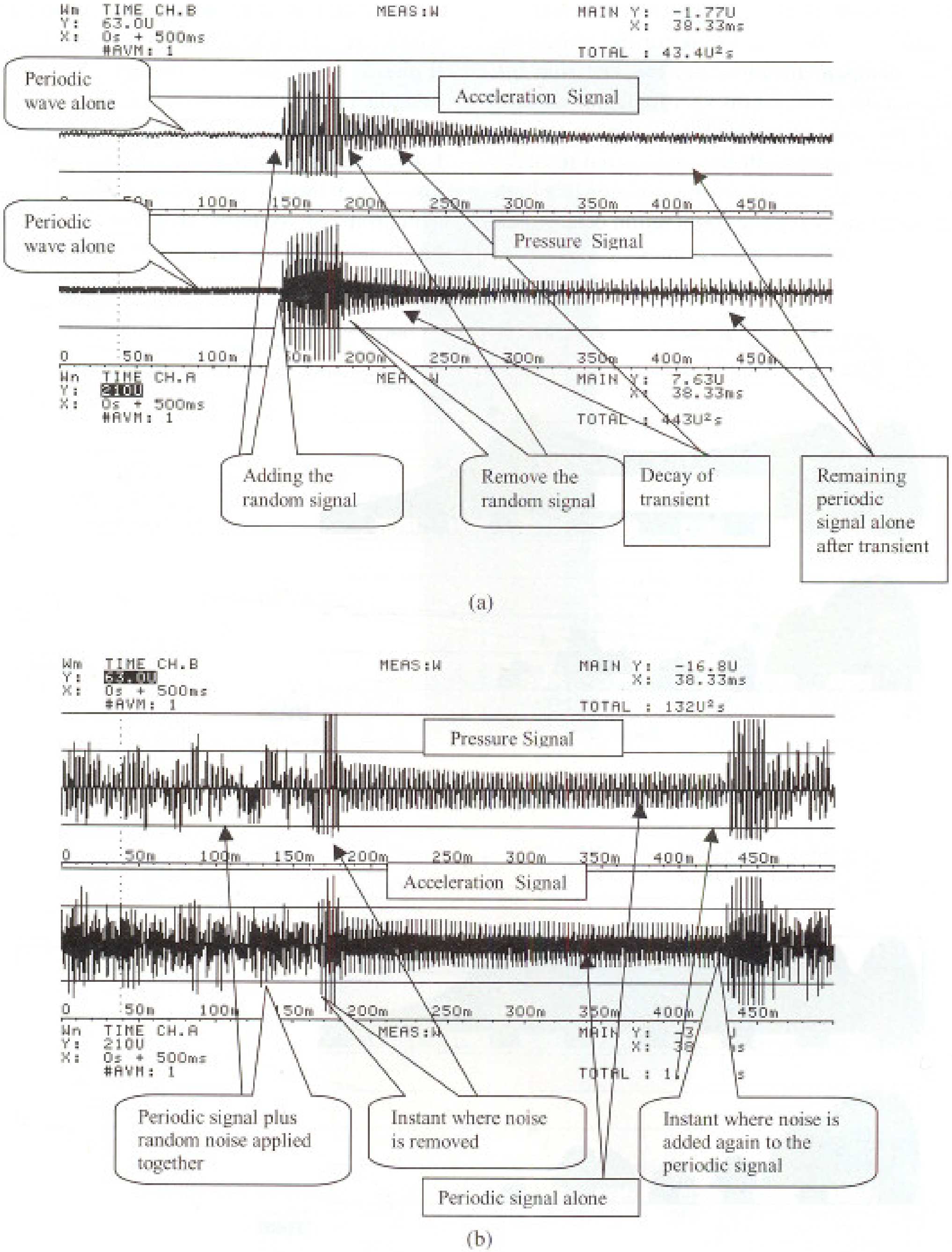

Fig. 8. Time-domain transient behavior. Magnitudes are acceleration signal in channel A; acoustic pressure in channel B. (a) Sequence is as follows: (1) We start applying the signal alone; (2) then we add the noise; (3) later we remove the noise, keeping the signal as it was at the beginning. Note that the remaining periodic signal is much higher than the one applied at the beginning of the sequence. (b) Sequence is as follows: (1) We start applying the signal plus noise; (2) then we remove the noise; (3) we apply the noise again. Note that each time we remove or add noise, we have a clear transient in the time signals, but the electronic adder is not free of transient contributions.

essential in order to average all blocks starting exactly at the same place. We also used a square wave as the clock signal that will trigger the spectrum analyzer.

Another point worth highlighting is that we used Nyquist's way of representing time, that is, the imaginary averaged part of the signal is graphed versus the real averaged part. Thus we can see the entire signal and need not limit ourselves to the real part. In general, this is a quite useful habit.

As usual, before anything else, we inject the rubbing loudspeaker with the periodic signal alone, the random signal alone, and then both signals simultaneously. According to what we explained earlier, when we average blocks of random signals using the periodic signal as a clock, our average naturally tends to zero. This means that each random signal block is uncorrelated to the other blocks with the same signal, and if we acquire several blocks and average them, this average will still tend to zero. The membrane's acceleration signal of the loudspeaker with rub and the pressure signal in the cavity also tend to average zero. We can then conclude with no surprise that the random signal is still random once it has been through the rubbing process. The average of the random signal is not shown in the figures because it would not make sense to show a blank result.

Fig. 9(a) illustrates the membrane's acceleration response (channel A) which carries more local or small phenomena information than the cavity pressure, which is more global. In the upper part of Fig. 9(a) we can observe the pressure in the cavity for the periodic signal alone (channel B). It must be taken into account that, as is normal, in a plane on paper the temporal axis is not visible. No notice should be taken of the fact that the line trace is not continuous. This is due to coincidences of the signal we are working with and the analyzer sampling.

Note that the acceleration signal forms loops. These loops are a form of amplitude modulation. Notwithstanding, the periodic signal has three loops centered on its ori-

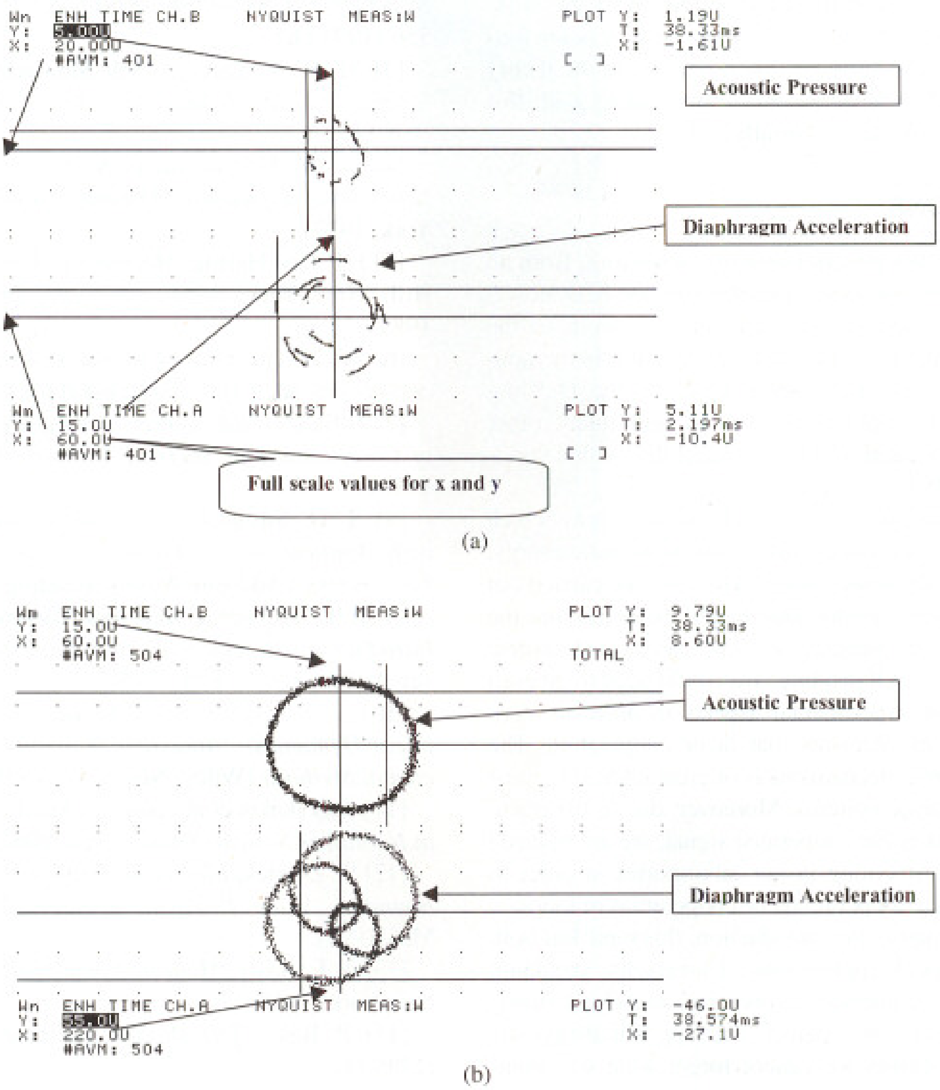

Fig. 9. Time-domain average using a square signal as a periodic one. Time signal is shown in complex form. Magnitudes are membrane acceleration in channel A; acoustic pressure in channel B. (a) Average of periodic signal alone. Observe the three loops in channel A and only one circle for the pressure averaged in the time domain. The loops of the acceleration membrane are almost concentric, with a small bias. Note that the acceleration signal is richer in contents than the acoustic pressure. Full-scale values are at left. (b) Average of periodic signal plus random noise, both applied together. Observe the high degree of cooperation. Note that secondary loops in the acceleration signal are shifted with respect to the periodic signal alone. Note also that the acceleration signal is richer than the acoustic pressure. Maximum values are at left.

gin. Fig. 9(b), in which the full pressure scale was raised from 5 to 15 units and the acceleration from 15 to 55 units, shows the results of averaging 400 time blocks with the two signals present simultaneously. Observe as well the good cooperation level obtained and how the cavity pressure has gone from 1.19 to 9.79 units (see cursor), a 9.15-dB increase.

As far as the membrane acceleration goes, besides an obvious cooperation phenomenon, the centers of the loops are not the center of the coordinates. We can predict from this a kind of low-frequency "carrier" against which our helix moves. Furthermore each of the four loops has a different center, and they are symmetrical in pairs.

Recall that what called our attention in the frequency domain were the even harmonics, and Fig. 9(b) seems to confirm this thesis for our system. We are referring to the fact that the harmonics found in spectrum averaging are even, since loops can be of an inferior frequency to that of the periodic excitation. What really happens is that this potential carrier has not manifested itself in the spectral-average analysis or has it remained hidden in the noise region of the cooperated signal. In any case, it is obvious that there are fine underlying synchronisms. In the theory of chaos a number of publications are dealing with this topic; see [15], [16], and, naturally, [1].

3 CONCLUSIONS

We have seen the phenomenon of cooperation from an experimental point of view. The theories we have nowadays, such as Bulsara and Gammaitoni [1], work on the hypothesis that the most plausible explanation is the nonlinearity in the potential energy term, or the Duffling oscillator (which explains the tunnel and many other important effects), and we have verified this with a variation of Van der Pol's oscillator.

We have generated a limit cycle in a system which instead of being supplied by direct current or static energy, was supplied by dynamic energy. The tests we carried out gave us satisfactory results. Due to the extension and the potentialities of the subject, we have performed diverse tests, but many are still pending. We would need to go even deeper into these experimental aspects in order to learn more about the mechanisms that favor cooperation. The knowledge of these mechanisms is of great interest to most branches of dynamic systems. Moreover, due to the phase lead we observed in the cooperated signal, we are induced to continue experimenting in our laboratories in order to bring about linked cooperations or cooperation in loops.

As we mentioned in the Introduction, this topic has both universal and local applications. Universally speaking, there is still a great unresolved question, namely, if strong nonlinearity seems to be a defect of nature, is it always so? In its local application we cannot forget what was mentioned at the beginning of this paper, referring to how in the field of electroacoustics there are still many open questions such as, why does an element with worse characteristics sound better than another one with better characteristics?

We could perhaps, then, conclude that these theories are valid to describe a reality which is somewhat wider than that formulated by Descartes. Nature's complexity requires also complex analyses and solutions.

4 ACKNOWLEDGMENT

The author would like to express his gratitude to J. Masip, director of Acústica Beyma, for his support. Thanks are also given to J. Martinez for his valuable advice. He would like to mention P. Seoane and K. Gronsbell, engineers from Beyma, who made his daily work much easier. Special appreciation goes to the reviewer of this *Journal* whose positive criticism and feedback encouraged the rewriting and improvement of this paper.

5 REFERENCES

[1] R. Bulsara and L. Gammaitoni, "Tuning in to Noise," *Physics Today* (1996 Mar.).

[2] C. Barratt, "On the Control of Chaos in Extended Structures," *J. Vibration and Acoust.*, vol. 119, pp. 551–556 (1997 Oct.).

[3] M. R. Schroeder, "Self-Similarity and Fractals in Science and Art," *J. Audio Eng. Soc.*, vol. 37, pp. 795–808 (1989 Oct.).

[4] D. W. Jordan and P. Smith, *Nonlinear Ordinary Differential Equations* (Oxford University Press, New York, 1977).

[5] J. P. Den Hartog, *Mechanical Vibrations* (McGraw-Hill, New York, 1956; rev. ed., Dover, New York, 1985).

[6] W. T. Thomson, *Theory of Vibration with Applications* (Prentice-Hall, Englewood Cliffs, NJ, 1982).

[7] J. Bendat and A. Piersol, *Engineering Applications of Correlation and Spectral Analysis* (Wiley, New York, 1993).

[8] S. H. Strogatz, *Nonlinear Dynamics and Chaos, with Applications to Physics, Biology, Chemistry and Engineering* (Addison-Wesley, Reading, MA, 1994).

[9] R. C. Hilborn, *Chaos and Nonlinear Dynamics, an Introduction for Scientists and Engineers* (Oxford University Press, New York, 1994).

[10] A. H. Nayfeh and B. Balachandran, *Applied Nonlinear Dynamics—Analytical, Computation, and Experimental Methods* (Wiley, New York, 1995).

[11] A. Vakakis et al., *Normal Modes and Localization in Nonlinear Systems* (Wiley, New York, 1996).

[12] W. Derrick, *Elementary Differential Equations with Boundary Value Problems* (Addison-Wesley, Reading, MA, 1997).

[13] T. L. Saaty, *Modern Nonlinear Equations* (Dover, New York, 1981).

[14] P. Bak, "The Devil's Staircase," *Physics Today* (1986 Dec.).

[15] F. Bolaños, "Nonlinear Systems and Chaos (in Spanish)," internal publ., School of Engineering La Salle, Barcelona, Spain (1990).

[16] C. Grebogi, E. Ott, and J. A. Yorke, "Crises, Sudden Changes in Chaotic Attractors and Transient Chaos," *Physica D*, vol. 7, pp. 181–200 (1983).

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